

# Robustness of $s$ -wave superconductivity against Coulomb interactions in $\text{Na}_x\text{CoO}_2$

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## Abstract

We study the depairing effect due to Coulomb interactions in  $\text{Na}_x\text{CoO}_2$ . We consider the electron-phonon coupling and the Coulomb interactions, and determine  $T_c$  for  $s$ -wave superconductivity by solving the linearized Eliashberg equation. When we consider shear phonons as well as breathing phonons,  $T_c$  is enhanced by Suhl-Kondo (SK) mechanism. Since SK mechanism is strong against Coulomb interactions,  $T_c$  remains finite even if strong Coulomb interactions cancel out the attractive force due to breathing phonons. The orbital degree of freedom is important to understand the mechanism of superconductivity in  $\text{Na}_x\text{CoO}_2$ .

*Key words:*  $\text{Na}_x\text{CoO}_2$ , valence-band Suhl-Kondo effect, shear phonon

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In  $\text{Na}_x\text{CoO}_2$ , although several LDA calculations suggest that there are hole pockets composed of  $e'_g$  orbitals. However, the top of the  $e'_g$  bands locate below (but very close to) the Fermi level in ARPES measurements. Theoretically, we have shown that a weak pseudogap behavior observed in susceptibility or density of states (DOS) can be explained only when  $e'_g$  hole pockets are absent[1]. The closeness of the top of  $e'_g$  bands to the Fermi level suggests that the orbital degree of freedom may play a important role in the superconductivity of  $\text{Na}_x\text{CoO}_2$ . We have shown that relatively high  $s$ -wave  $T_c$  is realized in  $\text{Na}_x\text{CoO}_2$  in case we consider  $E_{1g}$  phonons (shear phonons) as well as  $A_{1g}$  phonons (breathing phonons) since the interband transition of Cooper pairs are induced by shear phonons[2]. This enhancement of  $T_c$  due to shear phonons is understood as Suhl-Kondo (SK) mechanism where superconductivity is realized by the interband hopping of the cooper pairs. Even if the  $e'_g$  bands are valence bands, this mechanism is valid if the top of the  $e'_g$  bands locates near Fermi level. We call this mechanism of

superconductivity the valence-band SK mechanism. Thus, we inferred that  $\text{Na}_x\text{CoO}_2$  is an  $s$ -wave superconductor thanks to the orbital degree of freedom. However, it is not clear whether  $s$ -wave superconductivity can be realized in  $\text{Na}_x\text{CoO}_2$  since the strong Coulomb interactions would reduce  $T_c$ .

In this paper, we study the depairing effect due to Coulomb interactions in a multi-orbital system which describes the effective bands of  $\text{Na}_x\text{CoO}_2$ . In this model, we consider the direct Coulomb repulsion  $U$  and the pair hopping  $J$ . The depairing effect on the effective attractive force between electrons in the same orbital is given by  $U$ , and that for the interband transition of Cooper pairs due to shear phonons is caused by  $J$ . We calculate the reduction of  $T_c$ , and find that the SK effect due to the shear phonon is strong against Coulomb interactions.

The electron-phonon coupling between  $t_{2g}$  ( $a_{1g} + e'_g$ ) electrons and relevant optical phonon (breathing and shear phonons) are represented by four coupling constant  $a_1, a_2, b_1, b_2$  which is shown elsewhere[2]. We calculate  $T_c$  by solving the linearized Eliashberg

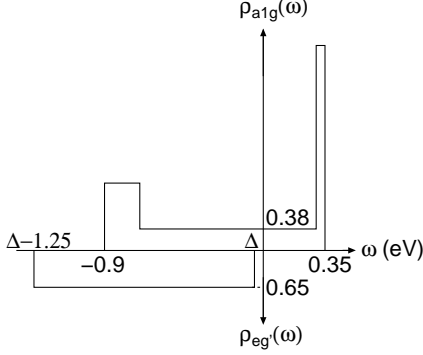


Fig. 1. DOS of  $\text{Na}_x\text{CoO}_2$ . The unit of energy is eV.

equation as follows.

$$\lambda \hat{\phi}(i\varepsilon_n) = T \sum_{i\varepsilon_n} \int_{-\infty}^{\infty} d\omega \frac{\hat{z}(\omega) \hat{\rho}(\omega)}{\varepsilon_n^2 + \omega^2} \hat{V}(i\varepsilon_n - i\varepsilon_{n'}) \hat{\phi}(i\varepsilon_{n'}), \quad (1)$$

$$\hat{V}(i\omega_n) = \begin{pmatrix} a_1^2 & 2b_1^2 \\ b_1^2 & a_2^2 + 2b_2^2 \end{pmatrix} D(i\omega_n) - \begin{pmatrix} U & 2J \\ J & U + J \end{pmatrix}, \quad (2)$$

where  $\hat{\phi}(i\varepsilon_n) = (\phi_{a1g}(i\varepsilon_n), \phi_{eg'}(i\varepsilon_n))$  is a column vector of gap functions.  $D(i\omega_n) = 2\omega_D/(\omega_D^2 + \omega_n^2)$  is the phonon's Green function, where  $\omega_D$  is the Debye frequency of phonon. We put  $\omega_D = 550$  ( $\text{cm}^{-1}$ ) for breathing and shear phonons.  $\varepsilon_n = (2n-1)\pi T$  and  $\omega_n = 2n\pi T$  are Matsubara frequencies. We show the expression of the renormalization factor  $z(\omega)$  for  $|\omega| < \omega_D$  elsewhere[3]. We put  $z(\omega) = 1$  for  $|\omega| > \omega_D$  for simplicity since the energy range where the band structure is renormalized is  $|\omega| < \omega_D$ . The DOS which we use in this calculation is shown in Fig. 1. This reproduces the bandwidth and the DOS near the Fermi level which is obtained by lattice model[1]. For  $a_{1g}$  band, we add the excess DOS in the band edge to reproduce the total number of  $d$ -electron and overall shape roughly. Note that the integration of DOS does not reach 1 because the ratio of the number of the  $d$ -electron in three  $t_{2g}$  bands near Fermi level is about 0.8 in the  $d$ - $p$  model.  $T_c$  is determined when  $\lambda$  in left-hand side of Eq. 1 is equal to 1.

Fig. 2 shows the  $U$ -dependence of  $T_c$  for  $J = U/10$ ,  $\Delta = -0.02$  (eV) and  $a_1 = a_2 = b_1 = 2b_2 = 0.23$  (eV). When we take account of only breathing phonons,  $T_c$  reduces rapidly and it vanishes at  $U = U_c = 2.8$  (eV). On the other hand, if we consider the shear phonons as well as breathing phonons,  $T_c$  remains finite even if  $U$  exceed  $U_c$ . This is because

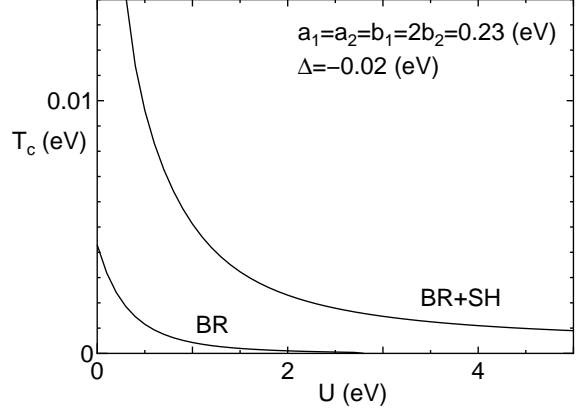


Fig. 2.  $U$ -dependence of  $T_c$  for  $J = U/10$ .

there is the effective attractive force due to inter-band hopping induced by shear phonons. This interaction is seldom weakened by Coulomb interactions since  $J \ll U$ . Thus, the  $s$ -wave superconductivity with  $T_c \sim 5\text{K}$  can be realized by this mechanism even if we consider the strong Coulomb interactions. In this calculation, the renormalization of  $U$  and  $J$  due to retardation is included automatically because the attractive force due to electron-phonon coupling depend on Matsubara frequency while repulsive force due to Coulomb interactions does not. We derive the renormalized  $U^*$  and  $J^*$  elsewhere[3]; it is shown that the renormalization for  $J$  is larger than that for  $U$ . Therefore, interband transition due to shear phonon is not prohibited by the Coulomb interactions.

In conclusion, we studied the depairing effect due to Coulomb interactions in  $\text{Na}_x\text{CoO}_2$ , and found that  $T_c$  remains finite even in the presence of the strong Coulomb interactions when we consider both breathing and shear phonons, while  $T_c$  vanishes rapidly when we consider breathing phonons only. Even if the direct Coulomb repulsion cancels out the attractive force induced by breathing phonons, the  $s$ -wave superconductivity due to SK mechanism would be realized. Thus, the superconductivity due to SK mechanism is realized even if the  $e'_g$  bands are valence bands.

## References

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